# Information Freshness and Packet Drop Rate Interplay in a Two-User Multi-Access Channel 

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#### Abstract

In this work, we combine the two notions of timely delivery of information to study their interplay; namely, deadlineconstrained packet delivery due to latency constraints and freshness of information. More specifically, we consider a twouser multiple access setup with random access, in which user 1 is a wireless device with a buffer and has external bursty traffic which is deadline-constrained, while user 2 monitors a sensor and transmits status updates to the destination. We provide analytical expressions for the throughput and drop probability of user 1. For user 2, we derive in closed form the age of information distribution, the average age of information (AoI), and the probability the AoI to be larger than a certain value for each time slot. The relations reveal a trade-off between the average AoI of user 2 and the drop rate of user 1: the lower the average AoI, the higher the drop rate, and vice versa. Simulations corroborate the validity of our theoretical results.


Index Terms-Age of information, deadline-constrained traffic, multi-access channel, random access.

## I. Introduction

THE proliferation of inexpensive devices with impressive sensing, computing, and control capabilities has lead to their widespread use in modern control environments which are often referred to as wireless networked control systems (WNCSs). Wireless sensors offer several advantages compared to their wired counterparts, such as scalability and flexibility in deployment at a lower cost, while at the same time they facilitate breaking new disruptive technologies into the market, such as autonomous vehicles, wireless industrial automation, environmental, and health monitoring, to name a few [2], [3]. In such scenarios, however, often a shared wireless network is used for information exchange, which introduces unique challenges that need to be addressed to avoid

[^0]degradation of performance or even loss of stability [4]. This setup accentuates the need for developing efficient algorithms offering timely delivery of information updates. On several occasions, this requires information to arrive at the destination within a certain period (deadline-constrained) due to stringent requirements in terms of latency, while in other cases, it is required to keep the information at the destination as fresh as possible. Information timeliness or freshness at the destination is captured by a new metric, called the age of information (AoI) [5], [6]. It was first introduced in [7], and it is defined as the time elapsed since the generation of the status update that was most recently received by a destination. In this work, we consider a two-user multiple access setup with heterogeneous traffic characteristics: one user with external bursty traffic which is deadline-constrained, while the other user monitors a sensor that transmits status updates, in the form of packets, to the destination; this is depicted in Fig. 1. The considered setup is expected to occur in several scenarios in wireless industrial automation (Industry 4.0), in which several processes are required to be completed within a predetermined timeline, and various processes are the preceding or the following of another and sensing the state of the system (or the preceding/following process) is essential.

## A. Related Works

Systems with deadline-constrained traffic have been considered almost two decades ago [8]. Packets with deadlines are connected with the notion of timely throughput, which measures the average number of successful deliveries before the deadline expiration. Recently, there has been a renewed interest in studying the performance of systems with deadline-constrained traffic [9], especially due to the ongoing automation of traditional manufacturing and industrial practices, under the fourth industrial revolution. For example, the works in [10]-[13] consider optimal scheduling schemes for traffic with deadlines. The works in [14], [15], study the performance of random access deadline-constrained wireless networks. In [16], [17], the authors analyze the benefits of scheduling based on exploiting variable transmission times in multi-channel wireless systems with heterogeneous traffic flows. In [18], the authors consider a joint scheduling-and-power-allocation problem of a downlink cellular system with real-time and non-real-time users. The authors proposed an algorithm that satisfies the hard deadline requirements for the real-time users and stability constraints for the non-real-time ones. In [19], a dynamic algorithm that solves the problem of


Fig. 1. An illustrative example of the system model. User 1 has deadlineconstrained bursty traffic, while user 2 monitors a sensor and its traffic is AoI-oriented.
minimizing packet drop rate in deadline-constrained traffic by optimizing power allocation under average power consumption constraints was proposed. The work in [20] considers a mixed type of traffic with deadline-constrained users and users with minimum throughput requirements. A dynamic algorithm is proposed for minimizing the packet drop rate while satisfying the throughput and average power consumption constraints. In [21], the authors consider a wireless system that includes users with buffers handling packets with deadlines and users with buffers handling queues with packet arrivals. In this context, the problem considered is to minimize the drop rate while guaranteeing queueing stability. The authors propose a dynamic scheduling algorithm by utilizing tools from Lyapunov optimization theory and Markov decision theory. Furthermore, there is a line of works that studies the optimal number of retransmissions of a packet before its deadline expiration [22]-[24].

Recently, the optimization and analysis of the average age of information have attracted a lot of attention; see, for example, [25]-[34]. In [25], the authors consider the link scheduling problem in a multiple users system under age of information constraints. In [26], the authors consider the age of information minimization problem in an Internet of things (IoT) system in which packets arrive with either random or fixed deadlines. In addition, some works consider the transmission of status updates that arrive randomly at the users, [28], [29], [31]. The goal of these works is to find an optimal scheduling policy to minimize the average age of information. Furthermore, the mechanism of generate-at-will has been considered in the literature. In these cases, the user can sample fresh information at will, [30]-[33]. It has been shown, that whenever this is possible, the age of information of the system can be improved as well as the management of the resources.

There is a line of papers that consider the interplay of AoI with throughput or latency. For example, in [35], the authors consider age of information-oriented users and users with random packet arrivals. The goal is to minimize the average age of information while guaranteeing queueing stability of the users with the random packet arrivals. The work in [36] derives optimal status updating policies for a system with a source-destination pair that communicates via a wireless link, whereby the source node is comprised of a buffer and serves two traffic flows, one that is AoI sensitive and one that is throughput oriented. In [37], the authors consider a system with an age of information-oriented user, and a
user with packets with deadlines. In this work, one user can transmit per time slot. The age of information minimization problem is considered under timely throughput constraints. In [38], the authors study the performance of a multiple access channel with heterogeneous traffic: One grid-connected node has bursty data arrivals and another node with energy harvesting capabilities sends status updates to a common destination. The work closer to our work is [39], in which the interplay between delay violation probability and average AoI in a two-user wireless multiple-access channel with multi packet reception (MPR) capability is studied. Nevertheless, the authors do not consider packets with deadlines and they do not discard a packet even if the delay is larger than a threshold. On the other hand, we consider a user with packets with deadlines, and therefore, we have to cope with a number of dropped packets. This changes fundamentally the problem and a different analysis approach is needed to be investigated.

## B. Contributions

In this work, we study the interplay of deadline-constrained traffic and the information freshness in a two-user random access channel with MPR capabilities. The deadline-constrained user has external bursty traffic modeled by a Bernoulli process, and the incoming packets are stored in its buffer. Each packet has a predefined deadline, by which if it has not been received by the destination then it is dropped from the system. The second user monitors a sensor and generates status updates at will in a timeslot. Even though this setup is small, it is tractable to analyze and it serves as a building block for more advanced setups.

The contributions of this work are the following.

1) For the deadline-constrained user, we provide the distribution of the waiting time of a packet and the expression for the drop rate.
2) For the age of information-oriented user, we provide the distribution of the age of information, the average age of information, and the probability the age of information to be larger than a value for each time slot.
3) We validate the accuracy of our analytical findings with simulations.
The results show a trade-off between the average AoI of user 2 and the drop rate of user 1: The lower the average AoI, the higher the drop rate, and vice versa. This is expected, since for reducing either the drop rate or the average AoI, the probability of transmission of the corresponding user should increase, causing interference to the other user.

## II. System Model

We consider two users transmitting their information in form of packets over a wireless fading channel to a receiver as shown in Fig. 1. Time is assumed to be slotted. Let $t \in \mathbb{Z}_{+}$ denote the $t$ th slot.
At each time slot $t$, a packet arrives in the buffer of user 1 with arrival probability $\lambda$. Each packet $j$ of user 1 has a deadline of $d_{j}$ slots since its arrival. Therefore, the packet must be successfully transmitted within $d_{j}$ slots; otherwise, it
is dropped and discarded from the system. For simplicity of exposition, we assume that $d_{j}$ is the same for all packets, i.e., $d_{j}=d, \forall j$, which is usually the case of packets that belong to the same traffic flow. User 1 attempts for transmission, when its buffer is non-empty, with probability $q_{1}$ at each time slot.

At each time slot $t$, user 2 , with probability $q_{2}$, samples "fresh" information and attempts to transmit it in form of a packet. We consider that the procedures of sampling together with transmission take one time slot. User 2 discards the sampled packet after the attempted transmission.

## A. Physical Layer Model

We consider that a packet from user $i$ is successfully transmitted to the receiver if and only if the signal-to-noise ratio or signal-to-interference-and-noise ratio (without or with interference, respectively) is above a certain threshold $\gamma_{i}$, i.e., $\operatorname{SINR}_{i} \geq \gamma_{i}$. Let $P_{\mathrm{tx}, i}$ be the transmit power of user $i$, and $r_{i}$ be the distance between user $i$ and the receiver. The received power, when user $i$ transmits, is $P_{\mathrm{rx}, i}=h_{i} s_{i}$, where $h_{i}$ is a random variable (RV) representing small-scale fading and $s_{i}$ is the received power factor. Under Rayleigh fading, $h_{i}$ is exponentially distributed [40]. The received power factor $s_{i}$ is given by $s_{i}=P_{\mathrm{tx}, i} r_{i}^{-\alpha}$, where $\alpha$ is the path loss exponent. When only user $i$ transmits, the successful transmission probability for user $i$ is given by $P_{i / i}=\exp \left(-\frac{\gamma_{i} \eta}{v_{i} s_{i}}\right)$, where $v_{i}$ is the parameter of the Rayleigh fading RV (i.e., $h_{i} \sim \operatorname{Rayleigh}\left(v_{i}\right)$ ), and $\eta$ is the noise power at the receiver. When both users transmit, the successful transmission probability for user $i$ is given by [41, Theorem 1]

$$
\begin{equation*}
P_{i / i, j}=\exp \left(-\frac{\gamma_{i} \eta}{v_{i} s_{i}}\right)\left(1+\gamma_{i} \frac{v_{j} s_{j}}{v_{i} s_{i}}\right)^{-1} \tag{1}
\end{equation*}
$$

where $j=i \bmod 2+1 .{ }^{1}$ Then, the service probability for user 1 is

$$
\begin{align*}
\mu_{1} & =q_{1}\left(1-q_{2}\right) P_{1 / 1}+q_{1} q_{2} P_{1 / 1,2} \\
& =q_{1}\left[\left(1-q_{2}\right) P_{1 / 1}+q_{2} P_{1 / 1,2}\right] \tag{2}
\end{align*}
$$

and for user 2 is

$$
\begin{align*}
\mu_{2} & =q_{2}\left(1-q_{1} \operatorname{Pr}\{Q>0\}\right) P_{2 / 2}+q_{2} q_{1}\left(\operatorname{Pr}\{Q>0\} P_{2 / 2,1}\right) \\
& =q_{2}\left[\left(1-q_{1} \operatorname{Pr}\{Q>0\}\right) P_{2 / 2}+q_{1}\left(\operatorname{Pr}\{Q>0\} P_{2 / 2,1}\right)\right] \tag{3}
\end{align*}
$$

respectively, where $Q$ is a random variable that indicates if the buffer of user 1 is empty ( $Q=0$ ), or non-empty ( $Q>0$ ). Then, the average success probability for user 1 and user 2 is

$$
p_{1}=\left(1-q_{2}\right) P_{1 / 1}+q_{2} P_{1 / 1,2},
$$

and

$$
p_{2}=\left(1-q_{1} \operatorname{Pr}\{Q>0\}\right) P_{2 / 2}+q_{1}\left(\operatorname{Pr}\{Q>0\} P_{2 / 2,1}\right)
$$

respectively.

[^1]

Fig. 2. The DTMC that models the AoI evolution.

## III. Average AoI Analysis and Distribution

In this section, we provide the analysis for the average and distribution of AoI. Age of information represents how "fresh" is the information from the perspective of the receiver. Let $A(t)$ be a strictly positive integer that depicts the age of information associated with user 2 at the receiver. The age of information evolution at the receiver is written as

$$
A(t+1)= \begin{cases}1, & \text { successful packet reception }  \tag{4}\\ A(t)+1, & \text { otherwise }\end{cases}
$$

We model the evolution of the AoI as a discrete-time Markov chain (DTMC). According to (4), at each time slot, the AoI drops to one in the case of successful packet reception from user 2. Otherwise, it increases by one. The Markov chain is described by $P_{j \rightarrow i}=\operatorname{Pr}\left\{X_{t+1}=i \mid X_{t}=j\right\}$, where $X_{t}$ denotes the value of $A(t)$ at the $t$ th slot. $P_{j \rightarrow i}$ represents the probability to transit to state $i$ given that the current state is $j$. The DTMC is shown in Fig. 2, where $\bar{\mu}_{2}=1-\mu_{2} .{ }^{2}$ When the system is in state $i, \forall i$, it can transit only to two possible states: a) To state 1 , if we have successful packet reception; b) to state $i+1$, otherwise. The transition matrix of the Markov chain is shown below

$$
\mathbf{P}_{A}=\left[\begin{array}{ccccc}
\mu_{2} & 1-\mu_{2} & 0 & 0 & \cdots  \tag{5}\\
\mu_{2} & 0 & 1-\mu_{2} & 0 & \ddots \\
\vdots & \vdots & \vdots & \ddots & \ddots
\end{array}\right]
$$

We denote the steady state distribution of AoI by $\pi^{A}=$ $\left[\pi_{1}^{A}, \pi_{2}^{A}, \cdots\right]$. To obtain $\pi^{A}$ we solve the following linear system of equations,

$$
\boldsymbol{\pi}^{A} \mathbf{P}_{A}, \boldsymbol{\pi}^{A} \mathbf{1}=1
$$

Using the balance equations, we obtain

$$
\begin{equation*}
\pi_{i}^{A}=\left(1-\mu_{2}\right)^{(i-1)} \mu_{2}, \forall i, \tag{6}
\end{equation*}
$$

which represents the probability the age of information to have the value of $i$. We can now obtain the expression for the average AoI by using the steady-state distribution. The average AoI is calculated as

[^2]

Fig. 3. The DTMC for which the deadline of packets is equal to 3 time slots, i.e., $d=3$.

$$
\begin{align*}
\bar{A} & =\sum_{i=1}^{\infty} \pi_{i}^{A} i=\sum_{i=1}^{\infty}\left(1-\mu_{2}\right)^{i-1} \mu_{2} i \\
& =\frac{\mu_{2}}{1-\mu_{2}} \sum_{i=1}^{\infty}\left(1-\mu_{2}\right)^{i} i  \tag{7}\\
& \stackrel{(a)}{=} \frac{\mu_{2}}{1-\mu_{2}} \frac{1-\mu_{2}}{\mu_{2}^{2}}=\frac{1}{\mu_{2}} \tag{8}
\end{align*}
$$

where $(a)$ follows by utilizing $\sum_{i=1}^{\infty} i c^{i}=c /(1-c)^{2},|c|<1$.
Furthermore, we calculate the probability of the age of information to be larger than a value, $x$, where $x \in \mathbb{Z}_{+}$. We have that
$\operatorname{Pr}\{A>x\}=1-\operatorname{Pr}\{A \leq x\}=1-\sum_{i=1}^{x} \pi_{i}^{A} \stackrel{(b)}{=}\left(1-\mu_{2}\right)^{x}$,
where (b) follows by utilizing $\sum_{i=0}^{n} c^{i}=$ $\left(c^{n+1}-1\right) /(c-1), c \neq 1$. $\operatorname{Pr}\{A>x\}$ is characterized as the age of information violation probability which is an important metric and it indicates us the probability the age of information to have a value larger than $x$ at each time slot.

## IV. Packet Drop Rate of User 1

In this section, we provide the expression for the drop rate of user 1 . We consider that if a packet from user 1 is not successfully transmitted because of channel errors, we have the option to retransmit it. In particular, we retransmit the packet until it is either successfully transmitted or its deadline has expired. As a result, the maximum number of retransmissions is $d-1$.

We use a DTMC to model the system. In particular, the states of the Markov chain represent the waiting time of the packet that is in the head of the queue (i.e., the packet that is first in the buffer). The number of states of the DTMC is equal to $d+1$. In Fig. 3, we depict an example of a DTMC for a system with $d=3$. The system is in state 0 if there is no packet waiting in the buffer. It transits to state 1 after the arrival of a packet. The packet experiences one slot waiting time right after its arrival because we consider the early departure - late arrival model. Therefore, the packet has the chance to be delivered in the next slot and so on. When the system is in state 1 , it transits to state 0 if the packet at the head of the queue has been successfully transmitted (with probability (w.p.) $\mu_{1}$ ) and
no packet arrived in the current slot (w.p. $\bar{\lambda}$ ). From state 1, it transits to state 2 (w.p. $\bar{\mu}_{1}$ ) when the packet is not successfully transmitted. Finally, it remains in state 1 when the packet that is in the head of the queue, is successfully transmitted and a new packet arrived in the current slot (w.p. $\mu_{1} \lambda$ ).
The system is in state 2 when the packet that is in the head of the queue has a waiting time that equals two slots. It remains in state 2 if the packet of the head is successfully transmitted in the current and a new packet arrived in the previous slot w.p. $\mu_{1} \lambda$. From state 2 , the state transits to state 1 only if we have a successful transmission and arrival in the current slot and no packet arrived in the previous slot (w.p. $\left.\lambda \bar{\lambda} \mu_{1}\right)$. It transits to state 0 , from state 2 , only if the packet is successfully transmitted and no packets arrived in the two previous slots (w.p. $\bar{\lambda} \bar{\lambda} \mu_{1}$ ). It transits to state 3 only if the packet is not successfully transmitted (w.p. $\bar{\mu}_{1}$ ) and since the waiting time is equal to three, the packet is dropped.
The system remains in state 3 only if a packet arrived three slots before the current slot (w.p. $\lambda$ ), therefore its waiting time equals to three slots. Since the deadline is equal to three slots, if any packet arrived at least three slots ago, either it had been transmitted or dropped. The system transits to state 1 only if a packet arrived in the current slot and no packets arrived in the two previous slots (w.p. $\lambda \bar{\lambda}^{2}$ ). It transits to state 2 only if a packet arrived in the previous slot and no packet arrived two slots ago (w.p. $\lambda \bar{\lambda}$ ). Finally, the system transits to state 0 only if no packets arrived in the three previous slots, w.p. $\overline{\lambda^{3}}$.

The transition probability matrix (row stochastic) of the Markov chain, depicted in Fig. 3, is shown below.

$$
\mathbf{P}=\left[\begin{array}{cccc}
\bar{\lambda} & \lambda & 0 & 0 \\
\mu_{1} \bar{\lambda} & \mu_{1} \lambda & \bar{\mu}_{1} & 0 \\
\mu_{1} \bar{\lambda}^{2} & \mu_{1} \lambda \bar{\lambda} & \mu_{1} \lambda & \bar{\mu}_{1} \\
\bar{\lambda}^{3} & \lambda \bar{\lambda}^{2} & \lambda \bar{\lambda} & \lambda
\end{array}\right]
$$

In general, the transition matrix of the Markov chain in the general case, where the deadline is $d$, is shown below


We denote by $\boldsymbol{\pi}=\left[\pi_{0} \pi_{1} \cdots \pi_{d}\right]$ the steady-state distribution of the Markov chain. To derive $\pi$, we solve the following linear system of equations $\boldsymbol{\pi} \mathbf{P}=\boldsymbol{\pi}, \boldsymbol{\pi} \mathbf{1}=1$. We observe that $\boldsymbol{\pi}$ is an eigenvector of $\mathbf{P}$. After applying eigenvalue decomposition we obtain the eigenvectors and eigenvalues of matrix $\mathbf{P}$. We find the eigenvector that corresponds to the eigenvalue that is equal to 1 . We normalize the elements of the eigenvector and we obtain $\pi$. Then, we calculate the drop rate as $\bar{D}=\pi_{d} \bar{\mu}_{1}$. In addition, we calculate the probability the buffer of user 1 to be non empty; $\operatorname{Pr}\{Q>0\}=1-\pi_{0}$. Therefore, all the terms in (7) are now known and the average age of information can be computed.


Fig. 4. $q_{2}=0.5, \lambda=0.8$. $q_{1}=0.1,0.2, \cdots, 1$.

## A. Discussion on the Lumpability of a DTMC

The DTMC in Fig. 3 can be an aggregated form of a twodimensional (2D) DTMC which takes into account the actions of user 1 as individual cases. The 2D DTMC is described by $P_{(i, j) \rightarrow(u, k)}^{2 \mathrm{D}}=\operatorname{Pr}\left\{X_{t+1}=u, Y_{n+1}=k \mid X_{n}=i, Y_{n}=j\right\}$, where $X_{t}$ and $Y_{t}$ denote the states of the action of user 2 and the waiting time in the queue for a packet of user 1, respectively. Note that $X_{t}$ can take either the value of one (if user 2 is active) or zero (if user 2 is silent). Note that $P_{(0, i) \rightarrow(0, j)}^{2 \mathrm{D}}=P_{(1, i) \rightarrow(0, j)}^{2 \mathrm{D}}$ and $P_{(0, i) \rightarrow(1, j)}^{2 \mathrm{D}}=P_{(1, i) \rightarrow(1, j)}^{2 \mathrm{D}}$, $\forall i, j$, because the action of user 2 in the previous slot does not affect the transition in the current slot. Therefore, $P_{(0, i) \rightarrow(0, j)}^{2 \mathrm{D}}+P_{(0, i) \rightarrow(1, j)}^{2 \mathrm{D}}=P_{(1, i) \rightarrow(0, j)}^{2 \mathrm{D}}+P_{(1, i) \rightarrow(1, j)}^{2 \mathrm{D}}, \forall i, j$. Let us consider a partition of the states that is defined as $\mathcal{A}_{j}=\{(0, j),(1, j)\}$. According to Theorem 6.3.2 [42, Theorem 6.3.2, Chapter 6], the Markov chain is lumpable with respect to the partition $\mathcal{A}=\left\{\mathcal{A}_{1}, \mathcal{A}_{2}, \cdots, \mathcal{A}_{d}\right\}$. Therefore, the DTMC in Fig. 3 is an equivalent Markov chain with the 2D Markov chain described above.

## V. Numerical and Simulation Results

In this section, we provide results that show the interplay between the packet drop rate of user 1 and average AoI of user 2 at the receiver. Also, we validate our analysis by comparing the analytical with the simulation results.

We consider different scenarios. However, at each scenario we consider that the users are located at distance $r_{i}=30 \mathrm{~m}$ from the receiver. The receiver noise power is $\eta=-50 \mathrm{dBm}$, and the path loss exponent is $\alpha=4$. Also, both users transmit with power $P_{1}=P_{2}=10 \mathrm{~mW}$.
In Figs. 4, and 5, we depict the interplay between the average AoI of user 2 and packet drop rate of user 1. In Fig. 4, we show the effect of the value of $q_{1}$ on the average AoI. We consider four different cases each one with different MPR capabilities. We denote that a receiver has strong MPR capabilities only if $\delta=P_{1 / 1} / P_{1 / 1,2}+P_{2 / 2} / P_{2 / 2,1}>1$, otherwise we consider that the receiver has weak MPR capabilities. For $\gamma=-5 \mathrm{~dB}$ and $-3 \mathrm{~dB}, \delta=1.5195$ and


Fig. 5. $q_{1}=0.5, \lambda=0.5 . q_{2}=0.1,0.2, \cdots, 1$.
1.3323, respectively. Therefore, the receiver has strong MPR capabilities. For $\gamma=0 \mathrm{~dB}$ and 1 dB , then $\delta=1$ and 0.8854 , respectively. Therefore, the receiver has weak MPR capabilities.

In Fig. 4, we consider that the sampling probability is $q_{2}=$ 0.5 . We obtain the drop rate and average AoI for different values of $q_{1}$. We observe that when the receiver has strong MPR capabilities, the access probability of user 1 does not significantly affect the average AoI (red line). Therefore, in this case, we obtain that the best strategy would be to allow both users to transmit at the same time. On the other hand, we observe that as the MPR capabilities become weak the access probability of user 1 significantly affects the average AoI (black and green lines).
In Fig. 5, we consider that the access probability of user 1, $q_{1}$, is fixed and equal to 0.5 . Also, the arrival rate is $\lambda=0.5$. In this scenario, we consider different values of the sampling probability. We observe that when the receiver has strong MPR capabilities (red line), we can significantly decrease the average AoI while keeping the drop rate low for user 1 (red line). However, as the MPR capabilities become weaker, the drop rate is affected by higher values of $q_{2}$. To give a more realistic example, let us consider that our goal is to keep the average AoI below 5 . For $\gamma=-5 \mathrm{~dB}$, we observe that we can achieve this target for sampling rate $q_{2}=0.3$ and drop rate is 0.17 . Thus, allowing both users to transmit is beneficial. For the case which $\gamma=1 \mathrm{~dB}$ (weak MPR capabilities), we observe that, in order to keep the average AoI below 5, we should increase the sampling probability $q_{2}$ to the value of 0.7 . However, the drop rate for user 1 is high and it is equal to 0.41 , i.e., almost half of the packets are dropped. In this case, a time sharing scheme will be more beneficial.
In Fig. 6, we show how the value $\operatorname{Pr}\{A>x\}$ changes for different values of $q_{2}$ and $\gamma$. As we increase the value of $q_{2}$ the probability decreases because user 2 samples and attempts for transmission more often. However, due to the error-prone channel, we observe that $\operatorname{Pr}\{A>x\}$ has higher values for the case of weak MPR capabilities. Note that this metric, i.e., $\operatorname{Pr}\{A>x\}$, is important when we study the AoI in a slot


Fig. 6. Probability the AoI of user 2 at the receiver to be larger than a value, $x$. $\lambda=0.5, q_{1}=0.5$. Strong $(\gamma=-3 \mathrm{~dB})$ and weak $(\gamma=0 \mathrm{~dB})$ MPR capabilities.


Fig. 7. The AoI distribution for $\gamma=-5 \mathrm{~dB}$ for different values of $q_{2}$.
by slot basis. More precisely, based on this metric, we know what is the probability the AoI to be greater than a value of $x$ at a time slot. Therefore, we can configure or optimize the system, e.g., the value of $q_{2}$, based on our targets.

In Figs. 7-10, we present the AoI distribution for the second user for various values of $\gamma$ and $q_{2}$. In the legend of each figure, we present also the average drop rate for the first user to have a more complete view of the system's performance. We have considered the case $q_{1}=0.5, \lambda=0.5$.
Figs. 7 and 8 depict the AoI distribution for the strong MPR case. In this case, for $q_{2}=0.9$, the values AoI with non-zero probability are concentrated in the lower regime (less than seven). In these figures, we observe that even when the arrival probability is high and the transmission probability for the first user is $q_{1}=0.5$, the system can sustain both users with acceptable performance.

Figs. 9 and 10 depict the AoI distribution of the second user for the case of weak MPR capability. As we mentioned earlier, interference is destructive in this case thus, we observe


Fig. 8. The AoI distribution for $\gamma=-3 \mathrm{~dB}$ for different values of $q_{2}$.


Fig. 9. The AoI distribution for $\gamma=0 \mathrm{~dB}$ for different values of $q_{2}$.
high values for the drop rates for the traffic of the first user.
In addition, we observe that the distribution of AoI takes non-negligible values ( $>0.03$ ) over a larger span, and also the lower values of AoI of the second user have smaller probabilities compared to the case of strong MPR capabilities. Even when $q_{2}=0.9$ which is a high sampling and transmit probability for the second user, and $\gamma=1 \mathrm{~dB}$, having an AoI of 10 timeslots can happen with a probability of approximately 0.035 . Thus, here a time sharing scheme can be beneficial.

## VI. Conclusions and Future Directions

In this work, we studied the interplay of deadlineconstrained packet delivery and freshness of information at the destination. More specifically, we considered a two-user multiple access setup with random access, in which user 1 is a wireless device with a buffer and has external bursty traffic which is deadline-constrained, while user 2 monitors a sensor and transmits status updates to the destination. We


Fig. 10. The AoI distribution for $\gamma=1 \mathrm{~dB}$ for different values of $q_{2}$.
provided analytical expressions for the throughput and drop probability of user 1 . We provided the distribution regarding AoI, the AoI delay violation probability and the average age of information of user 2 in terms of closed form expressions. We demonstrated that there exists a trade-off between the average AoI of user 2 and the drop rate of user 1. Our analytical findings are validated through simulations.

From our results it is evident that the probability of accessing the channel affects the performance of individual users as well as that of the overall system. Ongoing work focuses on optimizing the performance of such systems utilizing the frameworks of Markov decision Processes and Lyapunov optimization. Furthermore, larger and more general setups will be considered.

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[^1]:    ${ }^{1}$ We would like to emphasize that the analysis presented in this work is more general and it can be applied to other channel cases as long as we can obtain the values for the success probabilities.

[^2]:    ${ }^{2}$ For simplicity of exposition, given a probability of an event, denoted by $p$, we denote the probability of its complementary event by $\bar{p}=1-p$.

