Coding Advantage Optimization of Space-time-frequency Block Codes

Vahid Momenaei Kermani, Hossein Momenaee Kermani, and Alireza Morsali

Abstract: **In this article, we study the coding advantage of space-time-frequency block codes (STFBCs). Since in MIMO-OFDM coding there is the potential of exploiting space, time and frequency diversities and as the frequency tones of OFDM modulation are relatively substantial, the intricate structure of STFBCs makes it almost impossible to optimize the parameters of the code as the number of symbols increases in each block. Consequently, we investigate how to optimize the code parameters independently to reduce the computational complexity. Furthermore, code permutation is an important part of the STFBCs. However, due to elaborate structure of the STFBCs, it is very hard to calculate the optimum permutation. Therefore, we put forth a technique to decompose the coding advantage of the STFBCs. Using this technique, the permutation parameter can be easily introduced and optimized for STFBCs. We then design a novel STFBC based on any given STBC and apply the proposed scheme on the proposed STFBC to illustrate the application of this technique by designing the optimum permutation for the code. Simulation results confirm that the proposed STFBC outperforms the best existing STFBCs in the literature.**

Index Terms: **Coding advantage, digital wireless communications, fading channel, MIMO-OFDM, space-time-frequency coding.**

I. INTRODUCTION

WIRELESS communication is in fact one of the basis of information and communications technology (ICT). Looking around at any time of the day, one can see a cornucopia of tools and gadgets which are becoming primary means of everyday life that are all mainly or partially using digital wireless communications [1]. Over a decade ago, maybe the best example could be the cellular phone. However, nowadays, in addition to the great demand for mobile high speed internet connection which is and will be offered by LTE and 5G, there are many other related applications which need high speed wireless communications [2], [3].

For instance, different protocols of local wireless networks, i.e., IEEE 802.11 with technological name Wi-Fi, are such popular that are now impossible to be removed from our lives. With the advent of the new era of Internet, "Internet of things", now we can see that even the objects and tools are going to use the Internet to communicate and cooperate [4]. Consequently, one can see that there are still many doors to be opened in the field of digital wireless communications. Multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) was one of the most promising techniques of modern high speed wireless communications. Combining the powerful OFDM modulation with MIMO systems was an ingenious scheme which became a very useful tool for wireless communications. This technique is now used in many standards and protocols. For example, many Wi-Fi standards are now using MIMO-OFDM coding for Mbit data transmission [5], [6]. Recently, the advent of massive-MIMO has also brought new life to MIMO systems and widened a broad horizons for this field of research [7].

Space-time-frequency block codes (STFBCs) are one of the best ways of implementing MIMO-OFDM system. These types of coding schemes were first introduced at the end of 90's based on the extensive attention which was focused on the basics of this technique, i.e., space time block coding [8]–[18]. In addition to the spatial and frequency diversity, STFBCs have also the potential of taking the advantage of time diversity. The design criteria for STFBCs are provided in [19]–[21]. In [20], OFDM subcarriers are divided into smaller groups but such grouping does not reduce the diversity gain. In [19], [22] systematic designs for STFBCs are presented to achieve full-diversity STFBCs. A full-rate full-diversity space-frequency block code (SFBC) is proposed in [23], in which a permutation method is applied in order to maximize the code performance where both the delay and the power profiles (DPPs) of the channel are known at the transmitter. Effects of presence and absence of DPPs at the transmitter are studied in [24].

In [25], based on time domain channel statistics acquired from the Kronecker correlation, precoders for STFBCs were designed by means of a series of orthogonal random codes to spread the space-time code over several subcarriers to attain multidiversity gains, in order to enable multiuser parallel transmission. Space-frequency block codes (SFBCs) are special cases of STFBCs which only occupy one timeslot for constructing the code in expense of losing the time diversity. Vector OFDM is used in [26] to design SFBCs based on Alamouti-like STBC with fixed diversity order over zero forcing (ZF) detection.

Signal identification is a modern technique for intelligent wireless communications. Recently, in [27], first, an SFBC identification algorithm is developed and then, as the basis of the identification process, time-domain properties of the Alamouti-like SFBC are derived analytically.

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V. M. Kermani is with the Department of Mathematics, Kerman Branch, Islamic Azad University, Kerman, Iran, email: vmkermani@gmail.com.

H. M. Kermani is with the Department of Pure Mathematics, Faculty of Mathematics and Computer, Shahid Bahonar University of Kerman, Kerman, Iran, email: momenaee@uk.ac.ir.

A. Morsali is with the Department of Electrical and Computer Engineering, McGill University, Montreal, Quebec, Canada, email: alireza.morsali@mail.mcgill.ca.

V. M. Kermani is the corresponding author.

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Permutation of STFBCs can significantly change the coding advantage (CA) of the coding scheme. This is basically related to the characterization of the OFDM frequency response in the frequency selective (FS) channels. Typically, the number of OFDM frequency tones is much more than the number of taps of the FS channel. For instance, in the typical 2-ray 5 μsec equal power frequency-selective channel there are only two taps whereas the OFDM signal can occupy more than 128 tones (recently 1024 and 2048 are also used). For this example, 128 coefficients of the frequency response are linear combinations of the two taps; hence the frequency coefficients are highly correlated. It is a well-known fact that the more uncorrelated channel gains the better the performance of the system. Consequently, permutation of a code can significantly affect its performance as discussed in details in [28]–[32].

Consequently, as also discussed in [28], [29], permutation plays a critical role in designing STFBC and SFBC. However, since the structure of these codes is complicated, it is hard and sometimes computationally impossible to optimize the permutation of the codes. The key concept to facilitate the optimization problem is the fact that the permutation is mostly related to the channel; therefore, if the CA is decomposable into two parts such that one part is related to the channel and permutation parameter and the other one to the code structure, finding the optimum permutation becomes a feasible procedure.

Nevertheless, to the best of our knowledge, systematic guidelines for such decomposition of STFBCs are not studied in the literature. For the first time it was shown that the SFBC in [23] has a decomposable coding advantage that works exclusively for this code and comes with the expense of sacrificing the code performance due to transmitting by only one antenna per subcarrier. It is recently shown in [30] that quasi-orthogonal STFBCs (QOSTFBCs) has also a decomposable CA. We also studied the decomposition of CA for space frequency coding in [29].

In this paper, we present a decomposition technique for CA of the STFBCs. Using this scheme, we build a structure to decompose the CA of the codes into two parts which then can be used to optimize different parameters of codes with less computation complexity. The proposed decomposition is a generalization of the SFBC decomposition we studied in [29]. We further design a novel STFBC based on any given STBC. Then, on the basis of the proposed technique we optimize the permutation of the code. As will be shown, the results confirm that the proposed code offers more than 1 dB improvement over the best existing coding schemes in the literature.

The rest of the paper is organized as follows: In the next section, the basics of MIMO-OFDM system model are introduced. In section III we present the decomposition technique and the examples describing how it can be applied to the codes. This is followed by section IV in which we present a novel STFBC design and its optimization based on the decomposition scheme. Section V presents and discusses the simulation results and conclusion of the paper is expressed in the last section.

Notations: Capital boldface letters stands for matrices, and boldface lower case alphabet with a bar represents vectors. By $\lfloor x \rfloor$ we mean the greatest integer less than or equal to x. The element on the i^{th} row and the j^{th} column of matrix $\mathbf{A}$ is denoted by $\mathbf{A}(\mathbf{i},\mathbf{j})$. The i^{th} element of vector $\overline{\mathbf{a}}$ is indicated by $\overline{\mathbf{a}}(i)$. Superscripts $(\bullet)^T$, $(\bullet)^H$and $(\bullet)^*$ indicate transpose, Hermitian and complex conjugations, respectively. By $\circ$ and $\otimes$, we mean the Hadamard and the Tensor products, respectively. Notation $\mathbf{1}_{a\times b}$ stands for an $a \times b$ matrix of ones, $\mathbf{I}_a$ denotes an identity matrix of size $a \times a$, and $\mathbb{C}$ stands for the complex field. $diag(a_1,\ a_2, \cdots,\ a_n)$ represents a diagonal $n \times n$ matrix whose diagonal entries are $a_1,\ a_2, \cdots,\ a_n$.

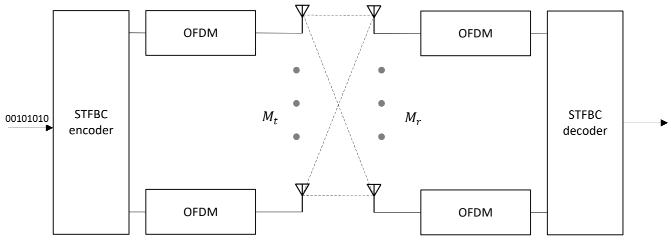

Fig. 1. STFB-coded MIMO-OFDM system with M_t transmit and M_r receive antennas.

II. SYSTEM MODEL

In this section, the system model of a MIMO-OFDM system is presented as shown in Fig. 1. We consider a MIMO-OFDM system with M_t transmit antennas and M_r receive antennas and N subcarriers within K successive OFDM blocks. The channel impulse response during the k^{th} OFDM block from the transmit antenna i to the receive antenna j is given by

$$h_{i,j}^{k}\left(\zeta\right)=\sum_{l=0}^{L-1}\alpha_{i,j}^{k}\left(l\right)\delta\left(\zeta-\zeta_{l}\right),\ k=1,\ 2,\cdots,\ K, \tag{1}$$

where ζ_l's are time delays and $\alpha_{i,j}^{k}(l)$'s are the complex amplitude of l^{th} path between the transmit antenna i and the receive antenna j which are modeled as zero-mean complex Gaussian random variables with variances $E\left|\alpha_{i,j}^{k}\left(l\right)\right|^2 = \sigma_l^2$. The condition $\sum_{l=0}^{L-1}\sigma_l^2 = 1$ is assumed to be held for normalization purposes. Assuming for different paths and different pairs of transmit and receive antennas the path gains are independent and the second-order statistics are the same, the time correlation at lag $m$can be defined as: $r_T(m) = E[\alpha_{i,j}^{k}\left(l\right)\alpha_{i,j}^{k+m}\left(l\right)^*]$. Thus, $\mathbf{R}_T$ is the $K \times K$ temporal correlation matrix if we let $\mathbf{R}_T\left(i,j\right) = r_T(i-j)$.

A STFBC is a $KN \times M_t$ matrix:

$$\mathbf{C}=\begin{bmatrix} \mathbf{C}_1^T & \mathbf{C}_2^T & \cdots & \mathbf{C}_K^T \end{bmatrix}^{\mathbf{T}} \tag{2}$$

with:

$$\mathbf{C}_k=\begin{bmatrix} c_1^k\left(0\right) & c_2^k\left(0\right) & \cdots & c_{M_t}^k\left(0\right) \\ c_1^k\left(1\right) & c_2^k\left(1\right) & \cdots & c_{M_t}^k\left(1\right) \\ \vdots & \vdots & \ddots & \vdots \\ c_1^k\left(N-1\right) & c_2^k\left(N-1\right) & \cdots & c_{M_t}^k\left(N-1\right) \end{bmatrix}. \tag{3}$$

where $c_i^k\left(n\right)$ is the signal to be transmitted on the n^{th} subcarrier from transmit antenna i during the k^{th} OFDM block. The transmitter applies an N-point inverse fast Fourier transform over each column of $\mathbf{C}_k$ and after adding cyclic prefix, the i^{th} column of $\mathbf{C}_k$ is transmitted by the transmit antenna i.

The received signal at antenna j in the k^{th} OFDM block, after the match filtering, having the cyclic prefix removed and implementing FFT at the n^{th} frequency subcarrier, is given by:

$$r_j^k(n) = \sum_{i=1}^{M_t} c_i^k(n) H_{i,j}^k(n) + \mathcal{N}_j^k(n), \quad n = 0, 1, \cdots, N-1. \tag{4}$$

where $H_{i,j}^k(n)$ is the channel frequency response at n^{th} subcarrier that is given by:

$$\mathcal{F}\{h_{i,j}^k(\zeta)\} = H_{i,j}^k(f)|_{f=n\Delta f} = H_{i,j}^k(n)$$

$$= \sum_{l=0}^{L-1} \alpha_{i,j}^k(l) e^{-j2\pi n\Delta f \zeta_l}. \tag{5}$$

The Fourier transform is denoted by $\mathcal{F}$, $\Delta f = 1/T_d = BW/N$. T_d is OFDM symbol period and BW is the total bandwidth. $\mathcal{N}_j^k(n)$ denotes the zero-mean additive white complex Gaussian noise with unit variance corresponding to the n^{th} frequency subcarrier at receive antenna j and the k^{th} OFDM symbol duration. Moreover, the frequency correlation matrix can be also defined as: $\mathbf{R}_F = E[H_{i,j}^k {H_{i,j}^k}^H]$, where $H_{i,j}^k = \left[H_{i,j}^k(0), \cdots, H_{i,j}^k(N-1)\right]^T$.

III. OPTIMIZING CA OF STFBCs

When we speak of the metrics for designing MIMO codes, typically we deal with the pairwise error probability (PEP). Since it is almost always impossible to determine the exact expression of PEP for these intricate coding schemes, a relatively close upper bound of the PEP is used. According to definitions of diversity order and coding advantage in [17], if for two distinct code words $\mathbf{C}$ and $\widetilde{\mathbf{C}}$, we have $P\left(\mathbf{C} \to \widetilde{\mathbf{C}}\right) \leq (CA\rho)^{-DO}$ where ρ is signal to noise power ratio (SNR), the exponent of ρ is known as the diversity order which determines the slope of the bit error curve at high SNRs and the coefficient of ρ is known as the coding advantage which for the codes with the same diversity order determines the enhancement of the codes.

The CA of a STFBC depends on both the code structure and the channel characteristics. In an L-ray channel, a STFBC could achieve the diversity order of rLM_t. where r is the rank of the temporal correlation matrix. Therefore, each of the LM_t rows of the matrix $\mathbf{C}_k$ must be correlated in order to achieve maximum available diversity. On the other hand, to minimize the complexity of the receiver whilst attaining maximum diversity, most existing STFBCs are designed by using sub-blocks of LM_t rows. Consequently, the construction of any STFBC could be broken down to design sub-blocks of size $\Gamma M_t \times M_t$, denoted by $\mathbf{G}_k^p$, for $p = 1, 2, \cdots, N/\Gamma M_t$, $1 \leq \Gamma \leq L$. Without loss of generality, we assume N is divisible by ΓM_t because otherwise the remaining subcarriers can be zero padded.

Let us assume each codeword of a STFBC be written as

$$\mathbf{C}_k = \begin{bmatrix} \mathbf{G}_k^1 \\ \mathbf{G}_k^2 \\ \vdots \\ \mathbf{G}_k^{N/\Gamma M_t} \end{bmatrix} \in \mathbb{C}^{N \times M_t}. \tag{6}$$

Define the primary codeword

$$\mathbf{G}_p = \begin{bmatrix} \mathbf{G}_1^p \\ \mathbf{G}_2^p \\ \vdots \\ \mathbf{G}_K^p \end{bmatrix} \in \mathbb{C}^{K\Gamma M_t \times M_t}. \tag{7}$$

The pairwise error probability between two distinct codewords $\mathbf{C}$ and $\widetilde{\mathbf{C}}$ is shown in [17] to be upper bounded by

$$P(\mathbf{C} \to \widetilde{\mathbf{C}}) \leq \binom{2vM_r - 1}{vM_r} \left(\prod_{i=1}^{v} \lambda_i\right)^{-M_r} \left(\frac{\rho}{M_t}\right)^{-vM_r}, \tag{8}$$

where λ_i's are the nonzero eigenvalues and v is the minimum rank of $\mathbf{\Delta} \circ \mathbf{R}$, respectively; where $\mathbf{\Delta} \triangleq \left(\mathbf{C} - \widetilde{\mathbf{C}}\right)\left(\mathbf{C} - \widetilde{\mathbf{C}}\right)^{\mathrm{H}}$ and $\mathbf{R} \triangleq \mathbf{R_T} \otimes \mathbf{R_F}$ with $\mathbf{R}_T$ and $\mathbf{R}_F$ denoting the temporal and frequency correlation matrices, respectively, which will be discussed in detail at the end of this section.

Now, from (6)–(8) for $\boldsymbol{G}_p$ we have:

$$P\left(\boldsymbol{G} \to \widetilde{\boldsymbol{G}}\right) \leq \binom{2\varsigma M_r - 1}{\varsigma M_r} \left(\prod_{i=1}^{\varsigma} \gamma_i\right)^{-M_r} \left(\frac{\rho}{M_t}\right)^{-\varsigma M_r}, \tag{9}$$

where γ_i's are the nonzero eigenvalues and ς is the minimum rank of $\widehat{\mathbf{\Delta}} \circ \widehat{\boldsymbol{R}}$, respectively, with $\widehat{\mathbf{\Delta}} \triangleq (\boldsymbol{G} - \widetilde{\boldsymbol{G}})(\boldsymbol{G} - \widetilde{\boldsymbol{G}})^{\mathrm{H}}$, and $\widehat{\boldsymbol{R}}$ is the principal sub-matrix of $\boldsymbol{R}$ with the same indexing as $\widehat{\mathbf{\Delta}}$ lies in the matrix $\mathbf{\Delta}$. Matrices $\boldsymbol{G}$ and $\widehat{\boldsymbol{G}}$ are two distinct primary codewords of size $K\Gamma M_t \times M_t$.

The diversity order of a STFBC is defined as vM_r and the minimum value of $\prod_{i=1}^{v} \lambda_i$ is known as the CA.

Using a similar structure for $\boldsymbol{G}_p$'s and taking two distinct codewords $\boldsymbol{C}$ and $\widetilde{\boldsymbol{C}}$, there is at least one index $p_0 (1 \leq p_0 \leq N/LM_t)$ such that $\boldsymbol{G}_{p_0}$ and $\widetilde{\boldsymbol{G}}_{p_0}$ are different. We may further assume that $\boldsymbol{G}_p = \widetilde{\boldsymbol{G}}_p$ for any $p \neq p_0$ since the rank of $\mathbf{\Delta} \circ \boldsymbol{R}$ does not decrease if $\boldsymbol{G}_p \neq \widetilde{\boldsymbol{G}}_p$ for some $p \neq p_0$. Thus, the rank and the CA of a STFBC are equal to those of the primary codeword described in (9), i.e. $v = \varsigma$ and $\min\left(\prod_{i=1}^{v} \lambda_i\right) = \min\left(\prod_{i=1}^{\varsigma} \gamma_i\right)$.

For full-diversity STFBCs with $\Gamma = L$, the matrix $\widehat{\mathbf{\Delta}} \circ \widehat{\boldsymbol{R}}$ is full-rank. Hence, all the eigenvalues of $\widehat{\mathbf{\Delta}} \circ \widehat{\boldsymbol{R}}$ are nonzero and we could calculate the CA by utilizing the following determinant:

$$CA = \min\left(\prod_{i=1}^{v} \gamma_i\right) = \min\left(\det\left(\widehat{\mathbf{\Delta}} \circ \widehat{\boldsymbol{R}}\right)\right), \quad v = LM_t. \tag{10}$$

From (10) it can be seen that the CA of STFBCs is changed by $\boldsymbol{R}$ which represents the correlation of the channel impulse response, the DPPs and the correlation of channel coefficients of different time slots. This phenomenon, therefore, results in different optimum structures of the codes in various channels. Hence, the structure of the codes achieving the optimum performance are different according to the characteristics of the channel.

The coding gain design criteria for STFBCs deals with $\widehat{\boldsymbol{\Delta}} \circ \widehat{\boldsymbol{R}}$ which is really hard to work with for different channels. Therefore, the following proposition is presented for decomposing the CA of any STFBC. Based on this decomposition, designing and optimizing the codes can be done much easier since the two parts of the decomposition correspond to the core structure of the code and the effect of the channel.

Similar to what we proposed for SFBCs in [29], let us now introduce a lower bound for the CA of full-diversity STFBCs, which decomposes the CA into two parts: *Core-*CA (C_{CA}) and *DPP-*CA (D_{CA}). Since SFBCs have lower dimension, the technique in [29] cannot be directly applied to STFBCs but the presented generalization in this paper is applicable to STFBCs and consequently to SFBCs and for $K = 1$ the techniques are the same. The *Core-*CA and the *DPP-*CA represent the effects of the precoder and the channel characteristics on the performance of the coding, respectively. The *Core-*CA can be used for precoder design and the permutation parameter of the STFBC and can be optimized with *DPP-*CA with much lower complexity than using the CA.

Proposition 1: (*Decomposition of STFBC's CA (DSC)*): For any full-diversity STFBC, if we have:

$$\widehat{\boldsymbol{\Delta}} = \widetilde{\boldsymbol{\Delta}} \circ \widehat{\Xi}, \quad (11)$$

where $\widetilde{\boldsymbol{\Delta}}$ is a positive semi-definite (PSD) matrix with nonzero diagonal entries and $\widehat{\Xi}$ is a PSD matrix, then the CA of SFBCs has a lower bound as follows:

$$CA \geq C_{CA}.D_{CA}, \quad (12)$$

where $C_{CA} \triangleq \prod_{i=1}^{v} \widetilde{\boldsymbol{\Delta}}(i,i)$ and $D_{CA} \triangleq \det\left(\widehat{\Xi} \circ \widehat{\boldsymbol{R}}\right)$.

Proof:

Substituting the assumption $\widehat{\boldsymbol{\Delta}}=\widetilde{\boldsymbol{\Delta}} \circ \widehat{\Xi}$ in (10) gives:

$$CA = \det\left(\widetilde{\boldsymbol{\Delta}} \circ \widehat{\Xi} \circ \widehat{\boldsymbol{R}}\right). \quad (13)$$

From results of [29] we know that $\boldsymbol{R}_F$ is a PSD matrix and letting $\boldsymbol{H}_T$ be the fading coefficient matrix, the temporal correlation matrix is calculated as $\boldsymbol{R}_T = \boldsymbol{E}[\boldsymbol{H}_T {\boldsymbol{H}_T}^H]$. Thus, $\boldsymbol{R}_T$ is also PSD and from the tensor product properties, we conclude that $\boldsymbol{R}$ is a PSD matrix as well.

The matrix $\widehat{\boldsymbol{R}}$ is also PSD, because any principal submatrix of a PSD matrix is PSD [33, p. 397]. Since the matrices $\widehat{\Xi}$ and $\widehat{\boldsymbol{R}}$ are PSD, the matrix $\widehat{\Xi} \circ \widehat{\boldsymbol{R}}$ is PSD again [33, p. 458]. Now, using the Oppenheim inequality [33, p. 480] for the Hadamard product, we can write:

$$\det(\widetilde{\boldsymbol{\Delta}} \circ \widehat{\Xi} \circ \widehat{\boldsymbol{R}}) \geq \prod_{i=1}^{v} \widetilde{\boldsymbol{\Delta}}(i,i) \times \det\left(\widehat{\Xi} \circ \widehat{\boldsymbol{R}}\right). \quad (14)$$

Defining $P_{CA} \triangleq \prod_{i=1}^{v} \widetilde{\boldsymbol{\Delta}}(i,i)$ and $C_{CA} \triangleq \det\left(\widehat{\Xi} \circ \widehat{\boldsymbol{R}}\right)$, we arrive at (12). □

To optimize parameters of a full-diversity STFBC by CA, calculation of CA is needed which requires calculating M^{N_s} determinants of $LKM_t \times LKM_t$ matrices, where M is the constellation size and N_s is the number of symbols used in each sub-block $\boldsymbol{G}_p$. For instance, for the full-rate STFBC we have $N_s = KL{M_t}^2$. The above proposition is useful as in such situations where the exact value of CA is not needed, and instead the lower bound of CA can be used which requires less complex calculations. For example, in order to obtain the optimum permutation for a full-diversity SFBC from N_γ possible permutations, using CA as the objective function requires calculation of $N_\gamma M^{N_s}$ determinants of $LKM_t \times LKM_t$ matrices, whereas with D_{CA}, it only requires calculating N_γ determinants. As a matter of fact, the computational complexity which is of order $\mathcal{O}(C \times M^{N_s})$ decreases to $\mathcal{O}(C)$, where $C = N_\gamma (LKM_t)^3$. Since the complexity reduction is substantial, the optimization process in the MIMO-OFDM systems would be very fast and easy.

One might suppose that the process of the proposed decomposition is arduous to perform. Thus, in what follows we show that the decomposition technique presented in the *Proposition* is easily applicable to some of the best STFBCs. We will also use this scheme to optimize our proposed STFBC in Section IV.

A. Examples of STFBCs

Example 1: In [22], considering the code structure, i.e. $\widehat{\boldsymbol{\Delta}}_{BCDD}$, we have $\widehat{\boldsymbol{\Delta}}_{BCDD} = \widetilde{\boldsymbol{\Delta}}_{\mathrm{BCDD}} \circ \widehat{\mathbf{I}}_{BCDD}$, where $\widehat{\Xi}_{BCDD}$ is a matrix whose entries are zero or one:

$$\widehat{\Xi}_{BCDD}(i,j) = \begin{cases} 1 & if\ \widehat{\boldsymbol{\Delta}}_{BCDD}(i,j) \neq 0 \\ 0 & if\ \widehat{\boldsymbol{\Delta}}_{BCDD}(i,j) = 0 \end{cases}, \quad (15)$$

meaning that $\widetilde{\boldsymbol{\Delta}}_{\mathrm{BCDD}}=\widehat{\boldsymbol{\Delta}}_{\mathrm{BCDD}}$.

Example 2: For the quasi-orthogonal space-time-frequency block codes (QOSTFBCs) [21], using the same technique, $\widehat{\boldsymbol{\Delta}}_{QOSTF} = \widetilde{\boldsymbol{\Delta}}_{QOSTF} \circ \widehat{\Xi}_{QOSTF}$, where $\widetilde{\boldsymbol{\Delta}}_{QOSTF} = \widehat{\boldsymbol{\Delta}}_{QOSTF}$, and

$$\widehat{\Xi}_{QOSTF}(i,j) = \begin{cases} 1 & if\ \widehat{\boldsymbol{\Delta}}_{QOSTF}(i,j) \neq 0 \\ 0 & if\ \widehat{\boldsymbol{\Delta}}_{QOSTF}(i,j) = 0 \end{cases}.$$

Example 3: For the STFBCs proposed in [32], we have $\widehat{\boldsymbol{\Delta}}_{[WSL]} = \widetilde{\boldsymbol{\Delta}}_{[WSL]} \circ \widehat{\Xi}_{[WSL]}$, where $\widetilde{\boldsymbol{\Delta}}_{[WSL]} = \widehat{\boldsymbol{\Delta}}_{[WSL]}$ and

$$\widehat{\Xi}_{[WSL]}(i,j) = \begin{cases} 1 & if\ \widehat{\boldsymbol{\Delta}}_{[WSL]}(i,j) \neq 0 \\ 0 & if\ \widehat{\boldsymbol{\Delta}}_{[WSL]}(i,j) = 0 \end{cases}.$$

Now, we show that the matrices $\widehat{\Xi}_{\mathrm{QOSTF}}$, $\widehat{\Xi}_{\mathrm{BCDD}}$ and $\widehat{\Xi}_{\mathrm{[WSL]}}$ are PSD. To this end, we use the fact that for any matrix $\mathbf{A}$, where $\mathbf{A}=\mathbf{D}\mathbf{D}^{\mathrm{H}}$, and $\mathbf{D}$ is an arbitrary matrix, $\mathbf{A}$ is a Hermitian PSD matrix.

For BCDD codes with $M_t = 2$, we have $\widehat{\Xi}_{\mathrm{BCDD}}=\mathbf{1}_{\mathrm{L}\times\mathrm{L}} \otimes \mathbf{I}_{2\times 2}$. It is easy to see that by defining $\mathbf{D}_{\mathrm{BCDD}}= (\mathbf{1}_{\mathrm{L}\times 1} \otimes \mathbf{I}_{2\times 2})$, we can write:

$$\widehat{\Xi}_{\mathrm{BCDD}}=\mathbf{D}_{\mathrm{BCDD}}{\mathbf{D}_{\mathrm{BCDD}}}^{\mathrm{H}},$$

proving that $\widehat{\Xi}_{\mathrm{BCDD}}$ is PSD. Also, for QOSTFBC with $M_t = 2$, $\widehat{\Xi}_{\mathrm{QOSTFBC}}$ has the same structure as $\widehat{\Xi}_{\mathrm{BCDD}}$ and therefore, it is

PSD. For the proposed code in [23], $\widehat{\Xi}_{[WSL]}=\mathbf{I}_{M_t\times M_t}\otimes\mathbf{1}_{L\times L}$, $\widehat{\Xi}_{[WSL]}=\mathbf{D}_{[WSL]}\mathbf{D}_{[WSL]}^{H}$, $\mathbf{D}_{[WSL]}=\mathbf{I}_{M_t\times M_t}\otimes\mathbf{1}_{L\times 1}$. This implies that $\widehat{\Xi}_{[WSL]}$ is PSD.

Since for all mentioned codes, $\widetilde{\boldsymbol{\Delta}} = \widehat{\boldsymbol{\Delta}}$ and $\widehat{\boldsymbol{\Delta}} = (\boldsymbol{G}-\widetilde{\boldsymbol{G}})(\boldsymbol{G}-\widetilde{\boldsymbol{G}})^{H}$, $\widetilde{\boldsymbol{\Delta}}$ is PSD for all of these SFBCs. Here, we obtained PSD matrices $\widetilde{\boldsymbol{\Delta}}$ and $\widehat{\Xi}$ for codes [21]–[23], which means the *proposition* is valid for these codes. One can see that for these SFBCs the matrix $\widehat{\Xi}$ is actually a binary mask drawn from the matrix $\widehat{\boldsymbol{\Delta}}$ and we do not need to find a special matrix $\widehat{\Xi}$.

IV. PROPOSED STFBC DESIGN BASED ON DCS

In this section, we present the structure of our STFBC design based on a given STBC. We should note that, any STBC with arbitrary number of antennas can be used using the same guideline we present; however, we use 2×2 STBC in [34], i.e., the Golden code to build a novel STFBC, namely GSTF. The Golden code for four symbols s_1, s_2, s_3, s_4 is constructed as:

$$\mathbf{F}_G(s_1,s_2,s_3,s_4)=\frac{1}{\sqrt{5}}\begin{bmatrix} a(s_1+bs_3) & a(s_2+bs_4)\\ jc(s_2+ds_4) & c(s_1+ds_3)\end{bmatrix}, \tag{16}$$

where $b=(1+\sqrt{5})/2$,$a=1+(j(1-b))$, $d=(1-\sqrt{5})/2$, and $c=1+(j(1-d))$ are the code parameters. We first design a novel STFBC and then DCS is used to decompose the code and find optimal permutation for the proposed STFBC.

Considering a MIMO-OFDM system with two transmit antennas and $\Gamma=L$ to achieve maximum possible frequency diversity. Using the same presentation as system model in section II, from (7), the sub-block $\boldsymbol{G}_p$ of the GSTF codeword can be written as:

$$\boldsymbol{G}_{p-GSTF}=\begin{bmatrix}\boldsymbol{G}_1^{pT} & \boldsymbol{G}_2^{pT} & \cdots & \boldsymbol{G}_K^{p\,T}\end{bmatrix}^T\in\mathbb{C}^{M_tKL\times M_t}, \tag{17}$$

where:

$$\boldsymbol{G}_k^p=\begin{bmatrix}\boldsymbol{X}_1^{kT} & \boldsymbol{X}_2^{kT} & \cdots & \boldsymbol{X}_L^{k\,T}\end{bmatrix}^T\in\mathbb{C}^{M_tL\times M_t}, \tag{18}$$

and in the case of the Golden code:

$$\boldsymbol{X}_m^k=\mathbf{F}_G\big(\overline{\boldsymbol{x}}_1(d_m^k+1),\overline{\boldsymbol{x}}_2(d_m^k+1),\overline{\boldsymbol{x}}_1(d_m^k+2),\overline{\boldsymbol{x}}_2(d_m^k+2)\big). \tag{19}$$

for $m=1,2,\cdots,L$ and $d_m^k=(k-1)LM_t+(m-1)M_t$ with

$$\overline{\boldsymbol{x}}_n{}^T=\boldsymbol{V}\overline{\boldsymbol{s}}_n{}^T, \tag{20}$$

where $\overline{\boldsymbol{s}}_n$ denotes a vector of M_tKL symbols taken from the constellation points, for $n=1,2,\cdots,M_t$ and $\boldsymbol{V}\in\mathbb{C}^{M_tKL\times M_tKL}$stands for the Vandermonde matrix whose parameters are selected in the same way as those in [23].

One should note that any other STBC can be used by just substituting the selected STBC in (19). Therefore, although $M_t=2$ in case of Golden code, all the equations and paramters are presented for the general case of M_t antennas.

Now, using DSC *proposition*, we can rewrite: $\widehat{\boldsymbol{\Delta}}_{GSTF}\circ\widehat{\boldsymbol{R}}$ as:

$$\widehat{\boldsymbol{\Delta}}_{GSTF}\circ\widehat{\boldsymbol{R}}=\widetilde{\boldsymbol{\Delta}}_{GSTF}\circ\widehat{\Xi}_{GSTF}\circ\widehat{\boldsymbol{R}}. \tag{21}$$

Due to particular structure of GSTF code, for practical constellations such as BPSK ,QPSK, 8PSK, 4-QAM, 16-QAM and 64-QAM , numerical results confirm that for any two distinct codewords $\boldsymbol{C}$ and $\widetilde{\boldsymbol{C}}$, the minimum of $\left(\det\left(\widehat{\boldsymbol{\Delta}}\circ\widehat{\boldsymbol{R}}\right)\right)$ occurs when either:

$$\boldsymbol{X}_m^k-\widetilde{\boldsymbol{X}}_m^k=\mathbf{F}_G\big(\overline{\delta}_1(d_m^k+1),0,\overline{\delta}_1(d_m^k+2),0\big) \tag{22}$$

or

$$\boldsymbol{X}_m^k-\widetilde{\boldsymbol{X}}_m^k=\mathbf{F}_G\big(0,\overline{\delta}_2(d_m^k+1),0,\overline{\delta}_2(d_m^k+2)\big), \tag{23}$$

where $\overline{\boldsymbol{\delta}}_n=\overline{\boldsymbol{x}}_n-\overline{\widetilde{\boldsymbol{x}}}_n$. Therefore, by introducing $\widehat{\Xi}_{GSTF}$ as follows:

$$\widehat{\Xi}_{GSTF}=(i,j)=\begin{cases}1 & if\ \widehat{\boldsymbol{\Delta}}_{GSTF}(i,j)\neq 0\\ 0 & if\ \widehat{\boldsymbol{\Delta}}_{GSTF}(i,j)=0\end{cases}, \tag{24}$$

we can write: $\widehat{\Xi}_{GSTF}=\mathbf{1}_{KL\times KL}\otimes\mathbf{I}_{\mathbf{2}\times\mathbf{2}}$ which is already shown to be PSD.

Now, since the conditions of the *proposition* are satisfied we can write:

$$CA_{GSTF}\geq\prod_{i=1}^{v}\widetilde{\boldsymbol{\Delta}}_{GSTF}(i,i)\times\det\left(\widehat{\Xi}_{\mathrm{GSTF}}\circ\widehat{\boldsymbol{R}}\right). \tag{25}$$

Now, in order to improve the performance of this SFBC, we can maximize its CA and this is done by maximizing the lower bound of (25). Therefore, we can maximize its D_{CA} by introducing a permutation parameter in the code.

Here, we present the permuted STFBC code ($PGSTF$):

$$\boldsymbol{C}_{PGSTF}=\boldsymbol{P}\boldsymbol{C}_{GSTF}, \tag{26}$$

where

$$\boldsymbol{P}=\mathrm{diag}(\overbrace{\widehat{\boldsymbol{P}},\ \widehat{\boldsymbol{P}}\cdots\widehat{\boldsymbol{P}}}^{\text{Number of}\widehat{\boldsymbol{P}}'\text{s}=\text{ K}})\in\mathbb{C}^{KN\times KN}, \tag{27}$$

and $\widehat{\boldsymbol{P}}=\mathrm{diag}(\overline{\boldsymbol{P}},\boldsymbol{I}_{\mathrm{a}})\in\mathbb{C}^{\mathrm{N}\times\mathrm{N}}$with:

$$\overline{\boldsymbol{P}}=\mathrm{diag}(\overbrace{\boldsymbol{Pb},\ \boldsymbol{Pb},\cdots,\ \boldsymbol{Pb}}^{\text{Number of}\boldsymbol{Pb}'\text{s}=\left\lfloor\frac{\mathrm{N}}{\mathrm{L}\gamma_{\mathrm{SD}}}\right\rfloor},\ \boldsymbol{P}'\boldsymbol{b})\otimes\boldsymbol{I}_{\mathrm{M_t}}. \tag{28}$$

In (28), γ_{SD} is the permutation parameter that will be indicated later, and $\boldsymbol{Pb}=\begin{bmatrix}\boldsymbol{P}_1^T & \boldsymbol{P}_2^T & \cdots & \boldsymbol{P}_L^T\end{bmatrix}^T\in\mathbb{C}^{L\frac{\gamma_{SD}}{M_t}\times L\frac{\gamma_{SD}}{M_t}}$, where:

$$\boldsymbol{P}_i=\begin{bmatrix}\boldsymbol{e}_i^T & \boldsymbol{e}_{L+i}^T & \cdots & \boldsymbol{e}_{(\frac{\gamma_{SD}}{M_t}-1)L+i}^T\end{bmatrix}^T\in\mathbb{C}^{\frac{\gamma_{SD}}{M_t}\times L\frac{\gamma_{SD}}{M_t}}, \tag{29}$$

for $i=1,\ 2,\cdots,\ L$, with $\boldsymbol{e}_j\in\mathbb{C}^{1\times L\frac{\gamma_{SD}}{M_t}}$ and $\boldsymbol{e}'_j\in\mathbb{C}^{1\times L\left\lfloor\frac{\gamma_r}{\mathrm{LM_t}}\right\rfloor}$are vectors whose components are all zeros except for the j^{th} element that is one, and:

$$\boldsymbol{P}'\boldsymbol{b}=\begin{bmatrix}\boldsymbol{P}_1'^T & \boldsymbol{P}_2'^T & \cdots & \boldsymbol{P}_{\left\lfloor\frac{\gamma_r}{LM_t}\right\rfloor}'^T\end{bmatrix}^T\in\mathbb{C}^{L\left\lfloor\frac{\gamma_r}{LM_t}\right\rfloor\times L\left\lfloor\frac{\gamma_r}{LM_t}\right\rfloor}, \tag{30}$$

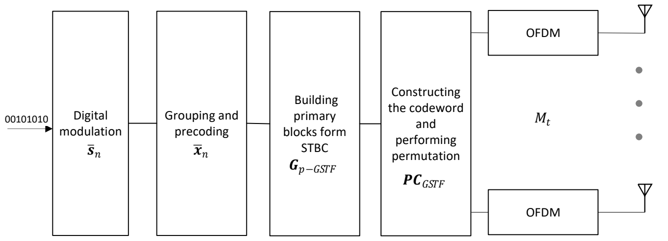

Fig. 2. Proposed STFBC encoder consisting of an STBC encoder and permutation.

where $\gamma_r = N - L\gamma_{SD}\left\lfloor \frac{N}{L\gamma_{SD}} \right\rfloor$ and for $i = 1,\ 2, \cdots,\ \left\lfloor \frac{\gamma_r}{LM_t} \right\rfloor$:

$$\boldsymbol{P'}_i = \begin{bmatrix} \boldsymbol{e}'^T_i & \boldsymbol{e}'^T_{\left\lfloor \frac{\gamma_r}{LM_t} \right\rfloor + i} & \cdots & \boldsymbol{e}'^T_{(L-1)\left\lfloor \frac{\gamma_r}{LM_t} \right\rfloor + i} \end{bmatrix}^T \in \mathbb{C}^{L \times L\left\lfloor \frac{\gamma_r}{LM_t} \right\rfloor}, \quad (31)$$

also $\boldsymbol{I}_a$ is an identity matrix for $a = \gamma_r - LM_t\left\lfloor \frac{\gamma_r}{LM_t} \right\rfloor$.

The modified code has an additional parameter, namely γ_{SD} which is an integer divisible by M_t, and can be optimized for variant channels. The block diagram of the STFBC encoder for the proposed scheme is shown in Fig. 2. Using the same guideline as (17) to (25), and from the *proposition*, we have:

$$CA_{PGSTF} \geq \prod_{i=1}^{v} \widetilde{\boldsymbol{\Delta}}_{PGSTF}(i,i) \times \det\left(\widehat{\boldsymbol{\Xi}}_{\mathrm{PGSTF}} \circ \widehat{\boldsymbol{R}}(\gamma_{\mathrm{SD}})\right), \quad (32)$$

where $\widehat{\boldsymbol{R}}(\gamma_{SD})$ is a submatrix of matrix $\boldsymbol{R}$ with the same indexes which $\widehat{\boldsymbol{\Delta}}_{PGSTF}$ lies in the matrix $\boldsymbol{\Delta}_{PGSTF}$. Finally, in order to optimize the positive integer γ_{SD}^{OP} to maximize the C_{CA} of $PGSTF$, the following optimization problem can be used:

$$\gamma_{SD}^{OP} = \arg \max_{1 < \gamma_{\mathrm{SD}} < \left\lfloor \frac{N}{L} \right\rfloor} \det\left(\widehat{\boldsymbol{\Xi}}_{\mathrm{PGSTF}} \circ \widehat{\boldsymbol{R}}(\gamma_{\mathrm{SD}})\right), \quad (33)$$

which can be easily solved by a simple search as γ_{SD}^{OP} is an integer less than $\left\lfloor \frac{N}{L} \right\rfloor$. For optimizing the permutation parameter γ_{SD}^{OP} with (33) one needs the correlation matrix $\boldsymbol{R}$ which can be constructed by DPPs.

It seems important to describe how the permuted STFBC affects the system at the receiver end. Since only one new parameter, namely γ_{SD}^{OP}, is introduced to the conventional structure of the STFBC; the only extra calculation at the receiver is the multiplication of the permutation matrix to the codeword before the ML decoder. Consequently, the complexity of the ML decoder does not change.

As discussed in section III, the DSC achieves optimization at low complexity. In order to illustrate this point, we consider an example of a practical MIMO-OFDM system. For a MIMO-OFDM transmitter with 2 transmit antennas, $K = 2$ and $N = 128$ subcarriers in the 6-ray TU channel model, optimization of the γ_{SD} parameter of the $PGSTFs$, using CA for 4-QAM constellation, calls for a lengthy calculation of $N_\gamma M^{N_s} = 10 \times 4^{8\times 6} \approx 10^{30}$ determinants of 12×12 matrices i.e. a complexity order of $\mathcal{O}(10^{36})$. In contrast, if the DSC is applied, then, to maximize the C_{CA} of $PGSTF$ only $N_\gamma = 10$ determinants of 12×12 matrices needs to be computed i.e. a complexity of the order of $\mathcal{O}(10^4)$.

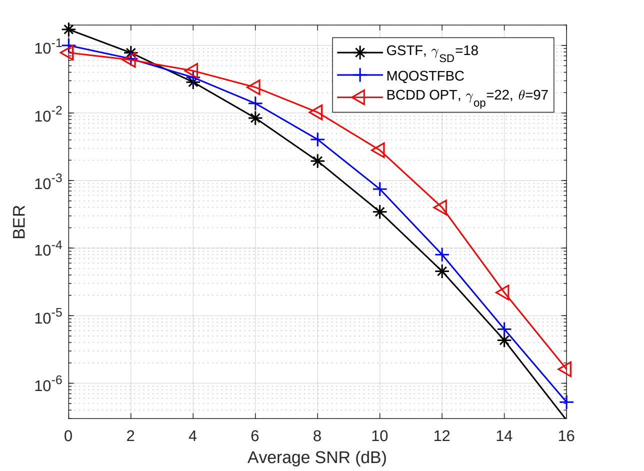

Fig. 3. BER performance, N=128, delay spread 20 μsec , BPSK for *PGSTF* and for BCDD OPT and QOSTFBC.

V. SIMULATION RESULTS

Let us consider a MIMO-OFDM system with two transmit antennas and two receive antennas, OFDM modulation BW = 1 MHz and a cyclic prefix of length 20 μs and a 2-ray equal power frequency-selective channel. It is further assumed that the transmitter has the knowledge of DPPs. In our simulations we presented the average bit-error-rate (BER) performance of our proposed code *PGSTF* and STFBC in [30], namely MQOSTFBC, the STFBC in [22], i.e., BCDD OPT and the SFBC in [29].

We first considered a MIMO-OFDM system with $N = 128$ subcarriers, $K = 2$ and a channel with 20 μsec delay spread. Fig. 3 shows BER versus the average signal-to-noise-ratio (SNR) of the *PGSTF*, MQOSTFBC and BCDD OPT. The parameters of the proposed optimum STFBC in [22] were calculated as: $\theta = 97$, $\gamma_{op} = 22$. For the *PGSTF*, the optimum value of γ_{SD} is equal to 18. In order to achieve the same spectral efficiencies, BPSK and QPSK constellations were used for *PGSTF* and codes in [22], [30] respectively. As can be seen our proposed STFBC outperforms both [30] and [22] .

We further considered a MIMO-OFDM system with $N = 64$ subcarriers and $K = 2$ and a frequency selective channel with 15μsec delay spread. In order to achieve the same spectral efficiencies, BPSK and QPSK constellations were used for *PGSTF* and BCDD OPT, QOSTFBC, respectively. Fig. 4 shows the simulation results for both the *PGSTF* and STFBCs in [22], [30]. The STFBC in [30] is the latest MIMO-OFDM code proposed in the literature to date, however, as can be seen *PGSTF* outperforms the QOSTFBC. For instance at BER 10^{-4} the proposed code outdoes the BCDD OPT and QOSTFBC by about 2 dB and 0.5 dB, respectively.

Finaly, we present the simulation result for $K = 1$ to show the advantage of the proposed code even as a SFBC in Fig. 5. We considered a system with 128 subcarriers and a channel with 5μsec delay spread. QPSK is used for PGSTF and $\gamma_{SD} = 14$ while 8PSK is used for QOSTFBC and BCDD OPT. The optimized parameters of BCDD OPT are: $\theta = 2$, $\gamma_{op} = 16$. The SFBC in [29] is also presented where its permutation parameter is equal to 12. It can be observed that the proposed STFBC

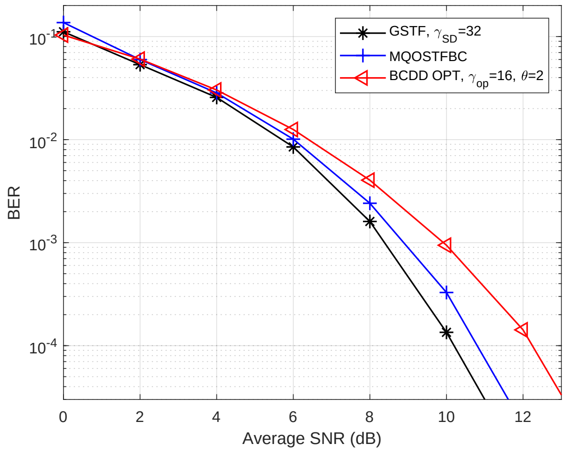

Fig. 4. BER performance, N=64, delay spread 15 μsec, BPSK for *PGSTF* and QPSK for BCDD OPT and QOSTFBC.

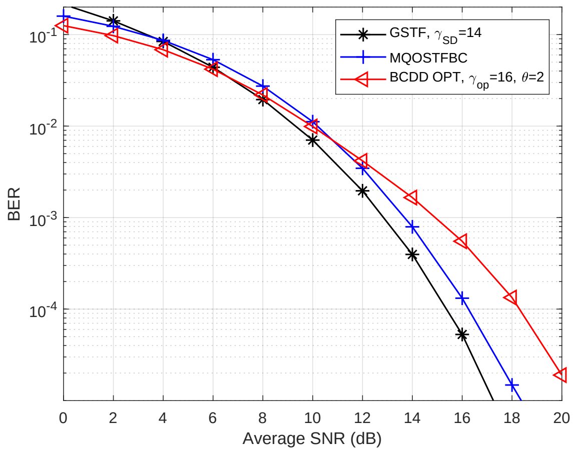

Fig. 5. BER performance, N=128, delay spread 5 μsec, QPSK for *PGSTF* and [29] and 8PSK for BCDD OPT and QOSTFBC.

has a superior performance compared to SFBCs in [30], [29], and [22].

VI. CONCLUSION

In this paper, we studied the coding advantage of STFBCs. STFBCs are usually designed to achieve the highest available diversity of the wireless system. However, they are not usually achieving the maximum coding advantage because the performance of the MIMO-OFDM codes highly depends on the characteristics of the channel. On the other hand, since STFBCs have complicated structures, it is very difficult to optimize the codes for various channels. In this paper, we first introduced a technique for decomposing the CA of the codes into two parts. Using this technique, one can optimize the permutation of the code to optimize the code for a specific channel with a very low complexity procedure. Moreover, the optimum precoder can be also derived using this scheme. Finally, a novel STFBC design was proposed based on any given STBC and further using the decomposition proposition, we presented the optimal permutation scheme for the newly designed STFBC. Simulation results also illustrate the superiority of the proposed code over best STFBCs in the literature.

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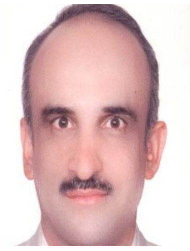

Vahid Momenaei Kermani received his BA in 1990 from Sharif University of Technology, Tehran, Iran in Electrical Engineering. Then he shifted to Mathematics and received his Ph.D. in 2010 from Azad University, Kerman Branch, Iran in Mathematics and is now a Faculty Member of the Mathematics Department in this university. His main research interests include optimization, operations research and mathematical modelling.

Hossein Momenaee Kermani received his M.Sc from Rutgers University, NJ, USA in 2001 and his Ph.D. from Shahid Bahonar University of Kerman, Kerman, Iran in 2005 both in Mathematics. He is now with the Department of Pure Mathematics, Shahid Bahonar University of Kerman, Kerman, Iran. His main research interests include space-time coding, matrix Analysis, and noncommutative algebra.

Alireza Morsali received his BA in 2009 and M.Sc in 2011 both from Shahid Bahonar University of Kerman, Kerman, Iran in Electrical Engineering. Since January 2016, he has been a Ph.D. student at McGill University, Montreal, QC, Canada. His main research interests include MIMO communications, coding theory, wireless communications and signal processing.